

Column Vectors & Magnitude

Patrons are reminded that the *magnitude* of a vector is its length. The magnitude of vector \mathbf{a} is denoted $|\mathbf{a}|$. It is found via Pythagoras:

$$\left| \begin{pmatrix} m \\ n \end{pmatrix} \right| \equiv \sqrt{m^2 + n^2}.$$

Find the values of the unknowns in the following equations:

$$1. \begin{pmatrix} p \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ q \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}. \quad \boxed{p = -1, q = -5}$$

$$2. \begin{pmatrix} 4 \\ w \end{pmatrix} = 3 \begin{pmatrix} p \\ -2 \end{pmatrix} - \begin{pmatrix} -5 \\ 4 \end{pmatrix}. \quad \boxed{p = -\frac{1}{3}, w = -10}$$

$$3. 5 \begin{pmatrix} -2 \\ \pi \end{pmatrix} - 2 \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} -1 \\ \frac{1}{2} \end{pmatrix}. \quad \boxed{m = -\frac{9}{2}, n = \frac{10\pi-1}{4}}$$

$$4. p \begin{pmatrix} 3 \\ -2 \end{pmatrix} + q \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad \boxed{p = -\frac{4}{13}, q = \frac{5}{13}}$$

$$5. m \begin{pmatrix} -1 \\ 4 \end{pmatrix} + n \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}. \quad \boxed{m = \frac{23}{22}, n = -\frac{13}{22}}$$

$$6. u \begin{pmatrix} 0 \\ 5 \end{pmatrix} + v \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -7 \\ -2 \end{pmatrix}. \quad \boxed{u = \frac{4}{25}, v = -\frac{7}{5}}$$

$$7. \left| \begin{pmatrix} p \\ 4 \end{pmatrix} \right| = 5. \quad \boxed{p = \pm 3}$$

$$8. \left| \begin{pmatrix} u \\ u+7 \end{pmatrix} \right| = 13. \quad \boxed{u = 5 \text{ or } u = -12}$$

$$9. \left| \begin{pmatrix} 2 \\ k \end{pmatrix} \right| + \left| \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right| = 8. \quad \boxed{k = \pm\sqrt{5}}$$

$$10. \left| \begin{pmatrix} 3m \\ m+3 \end{pmatrix} \right| + \left| \begin{pmatrix} 5 \\ -12 \end{pmatrix} \right| = 18. \quad \boxed{m = -\frac{8}{5} \text{ or } m = 1}$$

$$11. \left| \begin{pmatrix} m \\ m-1 \end{pmatrix} \right| + \left| \begin{pmatrix} 0 \\ m \end{pmatrix} \right| = 5. \quad \boxed{m = -4 \pm 2\sqrt{10}}$$

$$12. \left| \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right| + \left| \begin{pmatrix} -m \\ m+4 \end{pmatrix} \right| = 10. \quad \text{[Horrible numbers...]} \quad \boxed{m = \frac{-4 \pm \sqrt{194 - 40\sqrt{5}}}{2}}$$

$$13. k \begin{pmatrix} 2 \\ 1 \end{pmatrix} + m \begin{pmatrix} m \\ k \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}. \quad \text{[Quite hard.]} \quad \boxed{m = 0, k = 3 \text{ or } m = -3, k = -\frac{3}{2} \text{ or } m = 2, k = 1}$$

$$14. \left| \begin{pmatrix} t \\ 3 \end{pmatrix} \right| + \left| \begin{pmatrix} 1 \\ t \end{pmatrix} \right| = 8. \quad \text{[Hard.]} \quad \boxed{t = \pm \frac{3\sqrt{5}}{2}}$$